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CONTENTS

I. Introduction.

In this report we discuss the results obtained during the first six months of work under the NASA Grant NGR 10-007-005. Since most of the results obtained will appear with detailed proofs in mathematical periodicals, only brief discussions will be given here. One paper has been accepted by the *Bulletin of the American Mathematical Society*. Two other papers have been submitted for publication. A fourth paper was presented at a meeting of the American Mathematical Society, October 30th at the Massachusetts Institute of Technology. An abstract of this paper appears in the October, 1965 issue of *Notices of the American Mathematical Society*. Any reprints of these publications which are available by the next reporting date will be included as part of the progress report.

The personnel involved in this work and supported full time during the period June 1 through September 1, 1965 under this grant are Professor R. W. Bagley, Professor T. S. Wu and Mr. J. S. Yang, Ph.D. student in Mathematics. Professor Bagley is presently conducting a seminar with three Ph.D. candidates. The work discussed here is being studied in this seminar.

II. Recursive and Related Properties.

One of the principal objects under investigation at the beginning of the summer of 1965 can be described as follows: Let G be a topolcgical group and let G^* be the topological group obtained by assigning a new neighborhood base at the identity e of G consisting of all those sets $\bigcap_{t \in G} tVt^{-1}$ where $V \in \mathcal{U}$, a neighborhood base at e in the original topology of G. It is easy to see that G^* has equal left and right uniformities. As a matter of fact G^* has the smallest topology which contains the original topology of G under which G is a topological group with equal left and right uniformities. We have obtained several results concerning G^* but have encountered many questions which remain unanswered. One of the principal questions is: When is G^* locally compact? This question will be discussed in section III of this report. The reason for mentioning the question at this time is, while constructing an example of a locally compact topological group G for which G^* is not locally compact, a connection with the groups G^* and recursive properties was discovered. Specifically, it was discovered that a modification of the example referred to above could be used to solve a problem posed by W. H. Gottschalk in his paper, "Minimal Sets: An Introduction To Topological Dynamics," <u>Bulletin of the American Mathematical Society</u>, 1955, page 350. See 1) below.

The principal results of this section are as follows:

1) There exist locally compact groups with left almost periodic functions (in the sense of von Neumann) which are not right almost periodic (To appear in a paper in <u>Bulletin of the American Mathematical Society</u>.)

2) There exist connected Lie groups on which there are left uniformly almost periodic functions which are not right uniformly almost periodic and for these groups the set of all left uniformly almost periodic functions separate points. This result is the main result of a paper submitted to <u>Mathematica Scandinavica</u>.

3) A compact connected finite dimensional topological group which admits an expansive automorphism is Abelian.

4) Let Φ be the set of all real valued continuous functions on a locally compact topological group T, topologized with the compact-open topology. Let Δ_f be the closure in Φ of the set of right translation $\{f_t | t \in T\}$ of $f \in \Phi$. Let $p(g,t) = gt$, where $gt(\tau) = g(\tau t)$. Then (Δ_f, T, p) is an almost periodic minimal set if and only if f is bilaterally uniformly almost periodic.

III. <u>The Group G^*</u>.

In Section II we mentioned the question concerning local compactness of the group G^*. As a partial answer we have the following results:

1) If G is a locally compact group and H is a closed normal subgroup with equal left and right uniformities such that G/H is compact, then G^* is locally compact.

2) If G is a topological group and H is a normal subgroup such that G/H is compact and H^* is locally compact, then G^* is locally compact.

3) If G is a Lie group, then G^* is a Lie group.

4) If G is locally compact and is the closure of the group generated by the collection of all connected compact normal subgroups of G, then G^* is locally compact.

There is reason to conjecture that, if G is locally compact and connected, then G^* is locally compact. If this is true, then we would have the Corollary: If G is locally compact and G/G_o is compact, then G^* is locally compact. G_o is assumed to be the identity component of G.

IV. Structure of G^*.

The additive group of real numbers R is contained in $Gl(n)$ as a normal subgroup with the relative topology on R equal to the natural topology and R forms the center of $Gl(n)$, being the set of matrices λI where λ is a real number and I is the n by n identity matrix. If G is a topological group and R is contained in G as a normal subgroup, then R is contained in the center of G or intersects the center in the identity of G. Also, if G is a locally compact group containing R as a normal subgroup such that G/R is compact, then R is contained in the center of G.

Let H_V be the subgroup of G generated by V^* for each member V of a neighborhood base at e in G. Then $H = \bigcap H_V$ is a normal subgroup of G with equal left and right uniformities and G^*/H is totally disconnected. If G^* has only finitely many components, then $G^* = G$, i.e., G has equal left and right uniformities. If G^* is locally compact or if G is locally connected, then H is the identity component of G^*.

Let $G = Gl(n)$. The topology for G^* is determined by taking the smallest topology for G which contains the original topology and also the center as an open set. Thus, G^* is a one dimensional Lie group. Since the orthogonal subgroup H of G is compact, it has equal left and right uniformities. Thus, $H^* = H$. On the other hand there are subgroups H for which H^* is discrete. For example, the subgroup of all elements of G which have determinant equal to ± 1.

In addition to the question indicated in this report concerning locally compact connected groups several questions were discussed in a recent proposal to NASA for a continuation of support for research on this project.